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(6) VISCOUS FLOW OF ABLATING GREASE FILMS

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Nuclear/Chemical Pulse Reaction Propulsion Project

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VISCOUS FLOW OF ABLATING GREASE FILMS

Introduction

In a study of the ablation of metal surfaces by high-temperature gas flow, an experimental arrangement has evolved in which a gas of low density is driven down an evacuated tube at the end of which is a metal target plate of 12.5 cm diameter. The protective value of thin films of grease has come under study, and in this report we consider in turn the motion of the film under an assumed pressure profile, the motion under an assumed aerodynamic drag, and the decay of motion under no forces.

Without abandoning the more natural polar coordinates (as we have done) and without separating the problem into separate parts, T. Teichmann has taken an independent and more elegant, though perhaps less physically obvious, path to results which are in agreement with ours.

Grease Film Motion Produced by a Pressure Gradient

We consider the motion of a thin layer of viscous grease covering a flat stationary plate on which is impressed a pressure which varies with x. If the viscosity, μ, of the grease is sufficiently large, inertial forces may be neglected. Taking z as the coordinate perpendicular to the plate, whose surface is at $z = 0$, the equation

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{dp}{dx} \quad \left(= \frac{p'}{\mu}\right) \tag{1}$$

governs the dependence of u, the flow speed in the x-direction, upon z.

The boundary conditions for the solution of (1) are

$$u = 0 \text{ at } z = 0$$

$$\frac{\partial u}{\partial z} = 0 \text{ at } z = h$$

where h is the thickness of the grease layer. The second condition expresses the assumption that there is no drag force on the upper surface of the layer. Integrating (1) with these conditions gives

$$u(z) = \frac{p'}{2\mu}(z^2 - 2zh) \tag{2}$$

The material flux (per unit length in the y-direction) is then given by

$$F = \int_0^h u(z)dz = \frac{p'}{2\mu}\left(\frac{h^3}{3} - h^3\right) = \frac{-p'}{3\mu}h^3$$

Taking the grease to be incompressible, the equation of continuity may be written:

$$\frac{\partial h}{\partial t} = -\frac{\partial F}{\partial x} = \frac{1}{3\mu}\left(h^3 p'' + 3h^2 p' \cdot \frac{\partial h}{\partial x}\right)$$

or

$$\frac{\partial h}{\partial t} = \frac{h^2}{3\mu}\left(hp'' + 3p' \frac{\partial h}{\partial x}\right) \tag{3}$$

In writing this, we allow the film thickness to increase, but continue to regard velocities of flow in the z-direction as negligible.

Eq. (3) is separable. Let $h = X(x)\,T(t)$; then

$$XT' = \frac{1}{3\mu}X^2T^2(XTp'' + 3p'\,TX')$$

$$3\mu\frac{T'}{T^3} = X^2p'' + 3p'XX' = \lambda \tag{4}$$

where λ is the separation constant. The T-equation of (4) readily integrates to

$$T = \frac{1}{\sqrt{C - \frac{2}{3}\frac{\lambda t}{\mu}}}$$

The X-equation

$$X\frac{dX}{dx} + \frac{p''}{3p'}X^2 = \frac{\lambda}{3p'}$$

upon the substitution $v = X^2$ becomes

$$\frac{dv}{dx} + \frac{2p''}{3p'}v = \frac{2\lambda}{3p'}$$

with the standard solution

$$v = e^{-\frac{2}{3}\int\frac{p''}{p'}dx}\left\{\int\frac{2\lambda}{3p'}\,e^{\frac{2}{3}\int\frac{p''}{p'}dx}\,dx + K\right\}$$

Noting that $\int\frac{p''dx}{p'} = \log p'$, this can be simplified to

$$v = (p')^{-\frac{2}{3}}\left[\frac{2\lambda}{3}\int(p')^{-\frac{1}{3}}dx + K\right]$$

and

$$X = (p')^{-\frac{1}{3}}\left[\frac{2\lambda}{3}\int(p')^{-\frac{1}{3}}dx + K\right]^{\frac{1}{2}} \tag{6}$$

The solution to (3) is then

$$h = XT = (p')^{-\frac{1}{3}}\sqrt{\frac{\frac{2\lambda}{3}\int(p')^{-\frac{1}{3}}dx + K}{C - \frac{2}{3}\frac{\lambda t}{\mu}}}$$

Absorbing the quantity $\frac{3}{2\lambda}$ into the integration constants C and K gives

$$h = (p')^{-\frac{1}{3}}\sqrt{\frac{\int(p')^{-\frac{1}{3}}dx + K}{C - \frac{t}{\mu}}} \tag{7}$$

This is a "complete integral" of (3), and it remains to find a relationship between the constants C and K which will fit the initial conditions. Writing $C = f(K)$, a condition on f is*

$$\frac{\partial h}{\partial K} + \frac{\partial h}{\partial C}\frac{dC}{dK} = 0$$

or

$$\frac{dC}{dK} = -\frac{\frac{\partial h}{\partial K}}{\frac{\partial h}{\partial C}}$$

which becomes

$$\frac{dC}{dK} = \frac{C - \frac{t}{\mu}}{\int (p')^{-\frac{1}{3}}\,dx + K} = \frac{1}{(p')^{\frac{2}{3}}}\cdot\frac{1}{h^2} \qquad (8)$$

Now let the subscript zero denote values and functions at time zero, and introduce the notations

$$Q(x) = \int (p')^{-\frac{1}{3}}\,dx$$

$$S = \frac{dQ}{dx} = (p')^{-\frac{1}{3}}$$

Then (8) becomes

$$\frac{dC}{dK} = \frac{C - \frac{t}{\mu}}{Q + K} = \frac{S^2}{h^2} \qquad (9)$$

Setting $t = 0$ in (9) and inverting,

$$\frac{1}{\left(\frac{dC}{dK}\right)_o} = \frac{Q + K_o}{C_o} = \frac{h_o^2}{S^2} \qquad (10)$$

*See A. R. Forsyth, Theory of Differential Equations (Dover Publications, New York), Vol. V, Ch. 5, p.107.

It is to be borne in mind that $Q(x)$ and $S(x)$ are not boundary or initial conditions, but are representations of the pressure profile. The initial condition is $h_o = h(x, 0)$.

Differentiating (10) with respect to x gives

$$\frac{d}{dx}\frac{Q + K_o}{C_o} = \frac{\frac{dQ_o}{dx}\frac{dK_o}{dx}}{C_o} - \frac{Q_o + K_o}{C_o^2}\left(\frac{dC}{dK}\right)_o\frac{dK_o}{dx}$$

Substituting the expression (10) for $(dC/dK)_o$,

$$\frac{d}{dx}\frac{Q + K_o}{C_o} = \frac{\frac{dQ_o}{dx} + \frac{dK_o}{dx}}{C_o} - \frac{\frac{dK_o}{dx}}{C_o} = \frac{S}{C_o},$$

$$C_o = \frac{S}{\frac{d}{dx}\left(\frac{Q + K_o}{C_o}\right)} = \frac{S}{\frac{d}{dx}\left(\frac{h_o^2}{S^2}\right)} = \frac{S^4}{2\left(Sh_o\frac{dh_o}{dx} - h_o^2\frac{dS}{dx}\right)} \qquad (11)$$

From (10)

$$K_o = C_o\frac{h_o^2}{S^2} - Q = \frac{h_o S^2}{2\left(S\frac{dh_o}{dx} - h_o\frac{dS}{dx}\right)} - Q \qquad (12)$$

We now adopt the simplest initial condition and the one of most interest to us,

$$h(x, 0) = \delta, \text{ a constant}$$

and the simplest pressure profile of interest,

$$p(x) = p_o\left[1 - \left(\frac{x}{L}\right)^2\right]$$

Setting the constant $E \equiv L^{\frac{2}{3}} (2\, p_o)^{-\frac{1}{3}}$, we have

$$p' = \frac{2\, p_o}{L^2} x = -\frac{1}{E^3} x$$

$$p'' = -\frac{1}{E^3}$$

$$S = (p')^{-\frac{1}{3}} = -Ex^{-\frac{1}{3}}$$

$$\frac{dS}{dx} = \frac{1}{3} Ex^{-\frac{4}{3}}$$

$$Q = \int S dx = -\frac{3}{2} Ex^{\frac{2}{3}}$$

Then with $\frac{dh}{dx} = \frac{d\delta}{dx} = 0$, (11) and (12) become

$$\left.\begin{aligned} C_o &= -\frac{S^4}{2\delta^2 \frac{dS}{dx}} = -\frac{E^4 x^{-\frac{4}{3}}}{\frac{2}{3} Ex^{-\frac{4}{3}} \delta^2} = -\frac{3}{2} \frac{E^3}{\delta^2} \\ K_o &= -\frac{\delta S^2}{2\delta \frac{dS}{dx}} - Q = -\frac{E^2 x^{-\frac{2}{3}}}{\frac{2}{3} Ex^{-\frac{4}{3}}} + \frac{3}{2} Ex^{\frac{2}{3}} = 0 \end{aligned}\right\} \quad (13)$$

These are the values of the arbitrary constants in (7) which apply at time zero. The form (7) takes in the new notation is

$$h = -Ex^{-\frac{1}{3}} \sqrt{\frac{-\frac{3}{2} Ex^{\frac{2}{3}} + K}{C(K) - \frac{t}{\mu}}}$$

and if we suppose that the evaluation (13) might apply not only at time zero, but for subsequent time as well, i.e., that $C = C_o$ and $K = K_o$, this becomes

$$h = -E\sqrt{\frac{-\frac{3}{2} E}{-\frac{3}{2}\frac{E^3}{\delta^2} - \frac{t}{\mu}}} = -\frac{\delta}{\sqrt{1 + \frac{2\delta^2}{3\mu E^3} t}} = \frac{\delta}{\sqrt{1 + \frac{4 p_o \delta^2}{3\mu L^2} t}} \quad (14)$$

where in the last expression the minus sign has been absorbed by the ambiguity in the sign of the square root.

Eq. (14) satisfies the original differential Eq. (3) and the initial condition $h(0,x) = \delta$. The assumption that C and K are constants gives the correct final result because of the simplicity of the initial condition and the pressure profile. Returning to Eq. (3), it can in fact be seen by inspection that if $\partial h/\partial x$ is zero and p'' is independent of x, then $\partial h/\partial t$ will be independent of x, and the upper surface of the grease layer will remain a plane of height $h(t)$. Eq. (3) will then become a total differential equation whose solution is (14).

In a typical experimental situation,

$$p_o \cong 2 \text{ kb} = 2\times10^9 \text{ dynes/cm}^2$$

$$\delta \cong 0.001'' = 2.5\times10^{-3} \text{ cm}$$

$$\mu \cong 10 \text{ poises (for grease)}$$

$$L \cong 5 \text{ cm}$$

and (14) becomes

$$\frac{h}{\delta} = \frac{1}{\sqrt{1 + 64t}}$$

or

$$t = \frac{1}{64}\left[\left(\frac{\delta}{h}\right)^2 - 1\right] \quad (15)$$

This implies for example that the time required to reduce the thickness of the layer from one mil to one-fifth mil is

$$t = \frac{25 - 1}{64} = \frac{3}{8} \text{ sec}$$

The comparable time observed in the Orion ablation experiments is a thousand times shorter than this, and the conclusion therefore must be that the mechanism of viscous flow under the action of a pressure gradient is of negligible effect. (Considered the other way, the duration of the pressure pulse is roughly 2×10^{-4} sec. Under the conditions of the experiments, ablation of the order of 1/3% of initial thickness should occur in that length of time, whereas 80% ablation is observed.)

Grease Film Motion Produced by Aerodynamic Drag

If, instead of a pressure gradient p', a surface drag force D (dynes per sq.cm, say,) acts on the free surface of the film, the velocity profile is not parabolic as before but linear:

$$u(z) = \frac{D}{\mu} z$$

and the flux F is given by

$$F = \int_0^h u(z)dz = \frac{D}{2\mu} h^2 .$$

The equation of continuity now is

$$\frac{\partial h}{\partial t} = -\frac{\partial F}{\partial x} = -\frac{1}{2\mu} (h^2 D' + 2Dh \frac{\partial h}{\partial x})$$

which, in the light of the results above, we shall not treat as it stands but shall instead assume at the outset that h remains independent of x:

$$\frac{dh}{dt} = -\frac{D'}{2\mu} h^2 \qquad (16)$$

The solution to this ordinary differential equation will enable us to come quickly to a conclusion about the importance of surface drag in removing a grease film under the conditions of interest. This solution is

$$h = \frac{\delta}{1 + \frac{\delta D'}{2\mu} t} \qquad \text{or} \qquad t = \frac{2\mu}{\delta D'} \left(\frac{\delta}{h} - 1\right) \qquad (17)$$

where δ is the thickness of the grease layer at time zero.

To estimate the drag forces per unit area, it is necessary to estimate the flow speed, the viscosity coefficient, and the density of the gas which is assumed to produce the drag, and to take a characteristic length which will appear in the Reynolds number. A semi-empirical formula will then give the drag coefficient.

The flow speed will be taken more or less arbitrarily as one-tenth the average speed of the incoming flow, or about 3×10^5 cm/sec.

The viscosity coefficient of most gases lies between 0.7×10^{-4} and 2.5×10^{-4} poise at 273°K. Sutherlands' formula* for variation of viscosity with temperature predicts about an 8-fold increase at 10,000°K, so we take $\mu_{gas} = 10^{-3}$.

The gas density is about 10^{-3} gm/cc, and as a characteristic length we take 5 cm.

The Reynolds number of the flow is thus

$$R = \frac{VL\rho}{\mu} = \frac{3 \times 10^5 \times 5 \times 10^{-3}}{10^{-3}} \approx 10^6$$

and** in this regime the skin-friction drag coefficient is

$$C_D = \frac{0.072}{R^{\frac{1}{5}}} \approx 0.005$$

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*Jeans, Dynamical Theory of G[illegible] p 285

**Hoerner, Aerodynamic Drag, p.[illegible]

that is,

$$D = 0.005 \rho V^2 = 5 \times 10^{-3} \times 10^{-3} \times 9 \times 10^{10} = 5 \times 10^4 \text{ dynes/cm}^2$$

The derivative D' of the drag force is finally what is desired, and this may be taken to be the estimated drag force divided by the characteristic length, 5 cm. leaving $D' = 10^4$ dynes/cm^2/cm.

If, as before, we take $\delta = 2.5 \times 10^{-3}$ cm, $\mu = 10$ poises, and $\frac{\delta}{h} = 5$, then from (17)

$$t = \frac{2 \times 10 \times 4}{2.5 \times 10^{-3} \times 10^4} \approx 3 \text{ sec}$$

showing that aerodynamic drag, at least without Helmholtz instability, is even less important than the pressure gradient in the particular experimental conditions we are considering.

<u>Decay of Grease Film Motion Under No Forces</u>

In a few experiments the grease film was made much thicker, i.e., 30 mils instead of one mil. The resulting ablation left little more grease intact than was left from the thin films, and it is suggested that the "coasting" motion of the grease after the pressure and drag forces have been removed may be a contributing effect in what may be termed thick-film ablation. To find the manner in which the motion decays to rest, we write the equation*

$$\rho \frac{Du}{Dt} = \rho X - \frac{\partial p}{\partial x} + \mu \Delta^2 u \qquad (18)$$

which simplifies here to

$$\frac{\partial u}{\partial t} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial z^2} \qquad (19)$$

since 1) u is assumed independent of x and y, 2) there are no body forces, and 3) the pressure gradient . . . This is to be recognized as the

*Lamb, <u>Hydrodynamics</u>, p.577

heat conduction equation in one dimension with no sources, and with u and μ/ρ playing the roles of temperature and thermal conductivity, respectively. By imagining the flow to take place between two fixed planes 2h apart, the free surface is replaced by the median plane, the flow is unaltered, and the problem is the same as that of the decay of an initial distribution of temperature in a slab, the surfaces of which are maintained at zero temperature, a problem of great familiarity to readers of textbooks on Fourier series. For the initial distribution (2), the solution is*

$$u(z,t) = -\frac{8p'h^2}{\mu\pi^3} \sum_{m=1}^{\infty} e^{-\frac{m^2\mu\pi^2}{4\rho h^2}t} \frac{1+(-1)^{m+1}}{m^3} \sin\frac{m\pi z}{2h} \qquad (20)$$

of which the interesting part for our purpose is the leading term at $z = h$:

$$u(h, t) \simeq -\frac{8p'h^2}{\mu\pi^3} e^{-\frac{\mu\pi^2}{4\rho h^2}t} \qquad (21)$$

(p' appears here because it is a parameter of the initial motion.)

We take:

$h = 0.028'' \simeq 0.071$ cm; $h^2 = 0.005$

$p = 2 \times 10^9 \left[1 - \left(\frac{x}{L}\right)^2\right]$; $p' = -\frac{4x}{L^2} \cdot 10^9$

$L = 5$ cm; $x = 2.5$ cm; so, $p' = -4 \times 10^8$

$u = 10 \quad \rho = 1$

Then

$$u(h, t) = \frac{32 \times 10^5}{\pi^3} e^{-500\pi^2 t}$$

The distance through which the superficial layer moves before coming to rest is thus,

*Byerly, <u>Fourier's Series and Spherical Harmonics</u>, p.105

$$\ell = \int_0^\infty u\,(h, t)\,dt = \frac{32\times10^5}{\pi^3} \int_0^\infty e^{-500\pi^2 t}\,dt$$

$$= \frac{32\times10^5}{500\pi^5} \doteq 20 \text{ cm}$$

a distance which is meaningful in interpreting the thick-film ablation results. This distance goes as h^4, so for thin films it is not appreciable.

Ablative Flow of Layers of Molten Metal

In the absence of the protective coating of silicone vacuum grease which we have been considering, the hot gas interacts directly with the aluminum target plate, and recovered samples show that ablation occurs to a depth of about 0.015 cm, or about six times the thickness of an effective grease layer. This relative weakness of metal in resisting ablation is probably to be explained in large part by the very low viscosity coefficient of molten metal relative to grease. Most molten metals have viscosity coefficients in the range 1-2 centipoise.

The analysis of the flow of a melting layer of metal under the conditions of experimental interest is complicated by the interplay between thermal and dynamic processes and will be deferred.

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